

# Estimating the geocenter from GNSS data

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## A collinearity diagnosis of the GNSS geocenter determination

Paul Rebischung · Zuheir Altamimi · Tim Springer

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**Abstract** The problem of observing geocenter motion from global navigation satellite system (GNSS) solutions through the network shift approach is addressed from the perspective of collinearity (or multicollinearity) among the parameters of a least-squares regression. A collinearity diagnosis, based on the notion of variance inflation factor, is therefore developed and allows handling several peculiarities of the GNSS geocenter determination problem. Its application reveals that the determination of all three components of geocenter motion with GNSS suffers from serious collinearity issues, with a comparable level as in the problem of determining the terrestrial scale simultaneously with the GNSS

### 1 Introduction

Geocenter motion is usually defined, with varying sign conventions, as the relative motion between the center of mass of the total Earth system (CM) and the center of figure of the solid Earth surface (CF). Its geophysical cause is the redistribution of masses within the Earth system, from daily and sub-daily periods (e.g. ocean tides) to secular time scales (e.g. post-glacial rebound, present-day ice melting) via seasonal and inter-annual periods (e.g. water mass exchanges). As Earth satellites orbit around CM, geocenter motion affects the measurements of surface processes

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### Summary

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*It can be concluded without much exaggerating that current GNSS are insensitive to any component of geocenter motion.*

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# Estimating the geocenter from GNSS data

## Part I:

Stability of GNSS-derived Geocenter Estimates

## Part II:

Orbit Modelling Reflected by Geocenter Coordinate Series

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## Part I

# Stability of GNSS–derived Geocenter Estimates

# Stability of GNSS-derived Geocenter Estimates

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Description of the problem

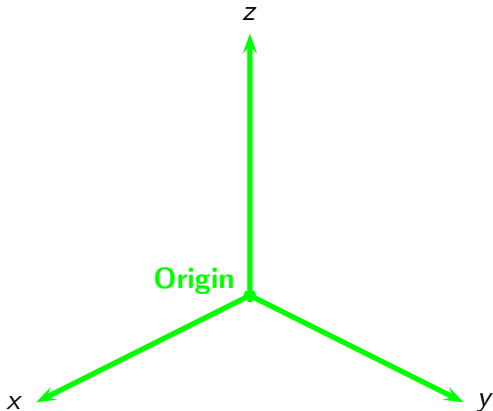
Experiment 1: Shifting the Geocenter

Experiment 2: Geocenter with Simulated Data

Geocenter Time Series from GNSS Solution

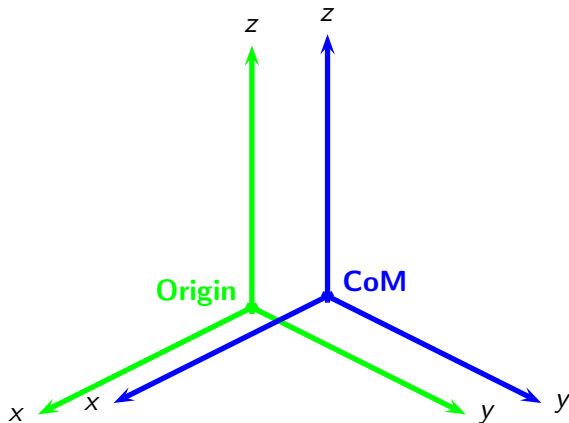
# Origin and Geocenter

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- Origin of the terrestrial reference system

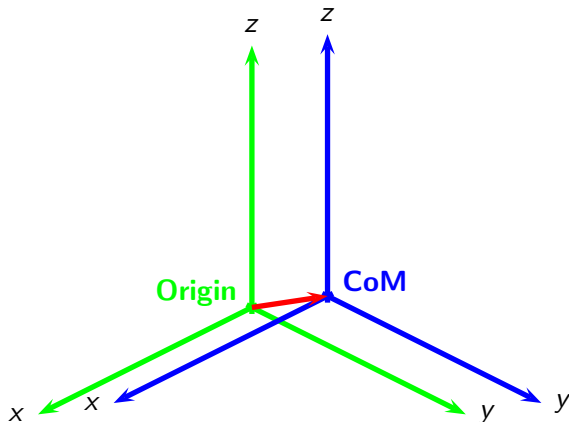
# Origin and Geocenter



- Origin of the terrestrial reference system
- Center of mass of the Earth

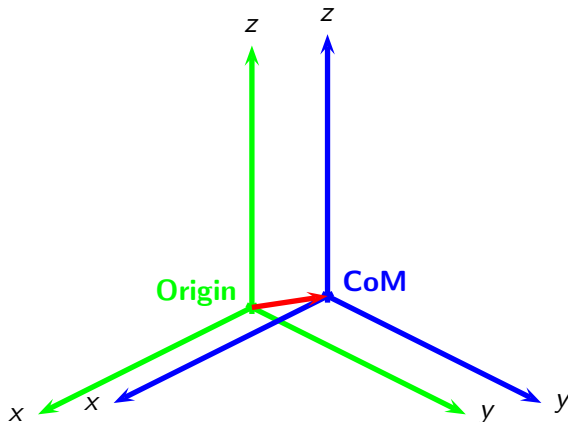


# Origin and Geocenter



- Origin of the terrestrial reference system
- Center of mass of the Earth
- Geocenter vector

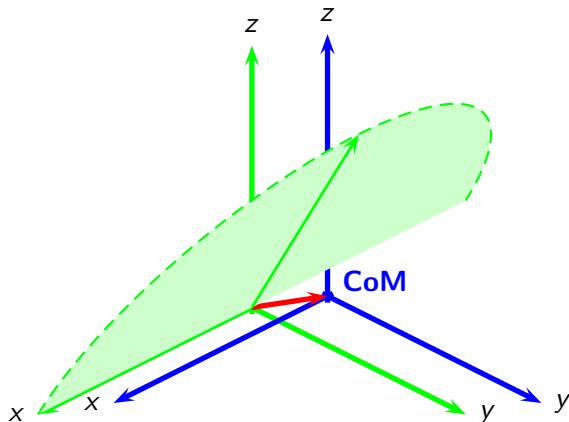
# Origin and Geocenter



- Origin of the terrestrial reference system
- Center of mass of the Earth
- Geocenter vector

The instantaneous **center of mass** differs from the long-term averaged location that is supposed to be the **origin of the terrestrial reference frame** by the **geocenter vector**.

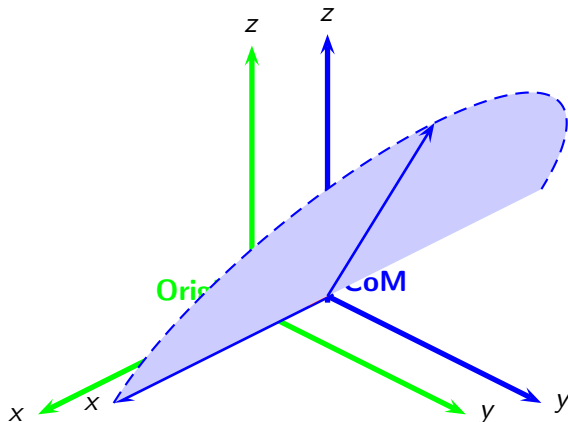
# Origin and Geocenter



- Origin of the terrestrial reference system
- Center of mass of the Earth
- Geocenter vector

The satellite orbit refers to the origin of the terrestrial reference system because the transformation from the terrestrial into the quasi-inertial system contains only rotations (Earth rotation parameters).

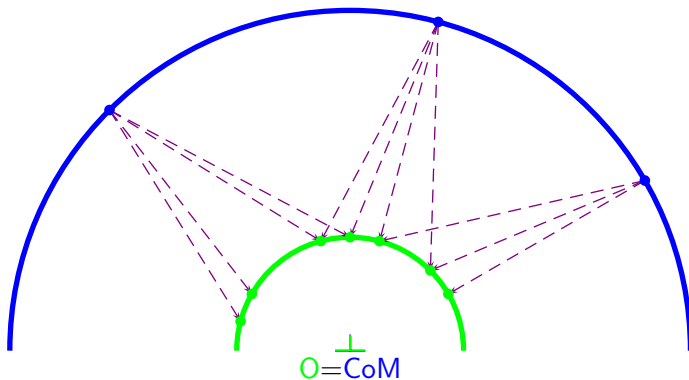
# Origin and Geocenter



- Origin of the terrestrial reference system
- Center of mass of the Earth
- Geocenter vector

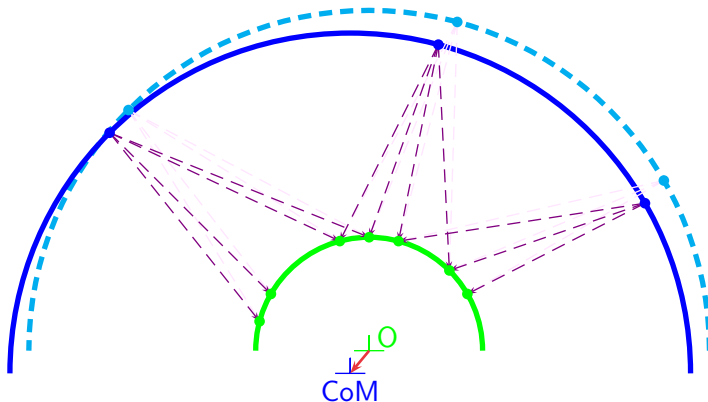
The satellite orbit refers to the center of mass of the Earth because the physics of celestial mechanics is based on the principle of gravitation.

# Consequences for the Data Analysis



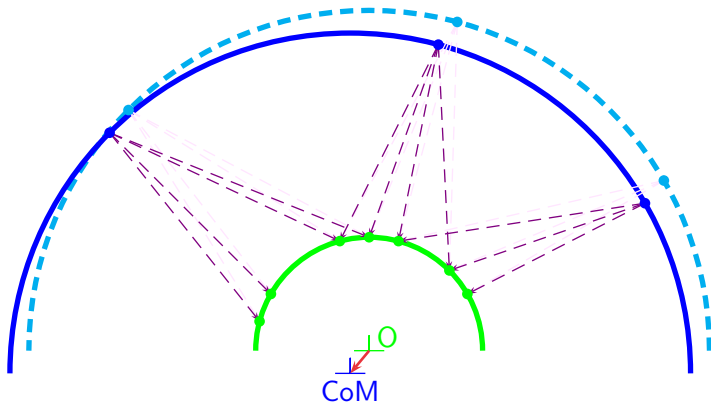
- In the processing model we typically assume that the **origin of the terrestrial frame** and the **center of mass** coincide in one and the same point.

# Consequences for the Data Analysis



- If this is not true (geocenter vector  $\neq 0$ ) we introduce an inconsistency between the processing model and the observations.

# Consequences for the Data Analysis



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*Are there parameters in the GNSS-analysis capable of absorbing this discrepancy?*

# Experiment 1: Shifting the Geocenter

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Parameters in the CODE–standard solution (GPS+GLONASS):

- **Orbit:** initial conditions, constant empirical SRP coefficients  $D_0$ ,  $Y_0$ ,  $X_0$ , once–per revolution for  $X$ –component; stochastic pulses at noon (constrained)
- **ERP:** offset and rates for polar motion and LOD; UT fixed
- **Troposphere:** vertical ZPD parameters every two; one set of gradient parameters per 24 hours
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- **Clocks:** implicit; epoch–wise independent



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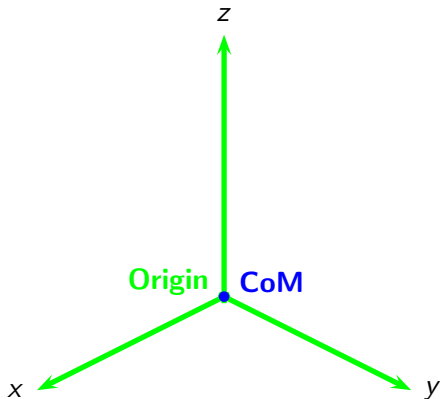
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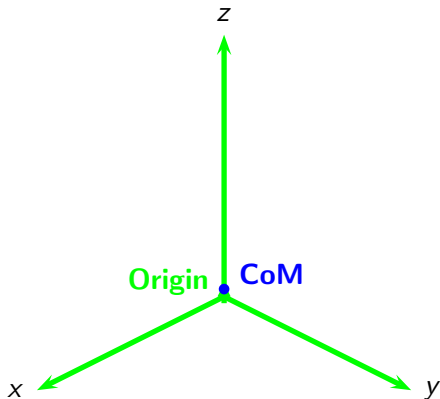
An arbitrarily chosen one–day solution has been selected for this experiment: January 21, 2014.

# Experiment 1: Shifting the Geocenter



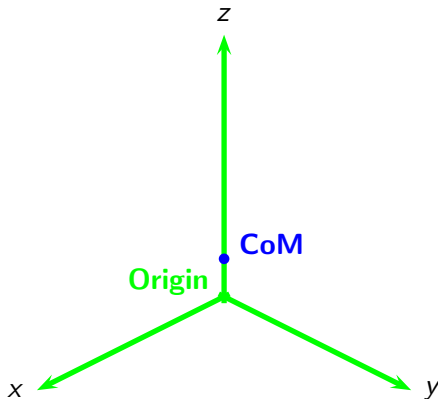
CoM wrt. Origin	RMS of adjustment
$(0,0,0.00)$ m	1.59 mm

# Experiment 1: Shifting the Geocenter



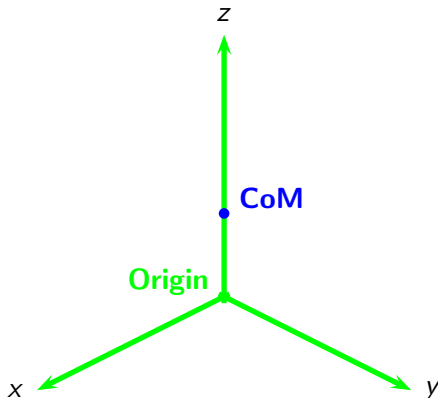
CoM wrt. Origin	RMS of adjustment
(0,0,0.00) m	1.59 mm
(0,0,0.01) m	1.59 mm

# Experiment 1: Shifting the Geocenter



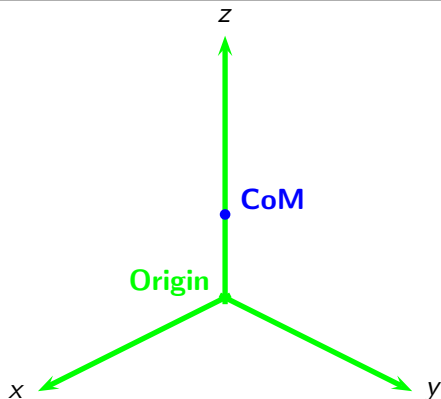
CoM wrt. Origin	RMS of adjustment
(0,0,0.00) m	1.59 mm
(0,0,0.01) m	1.59 mm
(0,0,0.10) m	1.60 mm

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CoM wrt. Origin	RMS of adjustment
(0,0,0.00) m	1.59 mm
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(0,0,0.10) m	1.60 mm
(0,0,1.00) m	2.01 mm

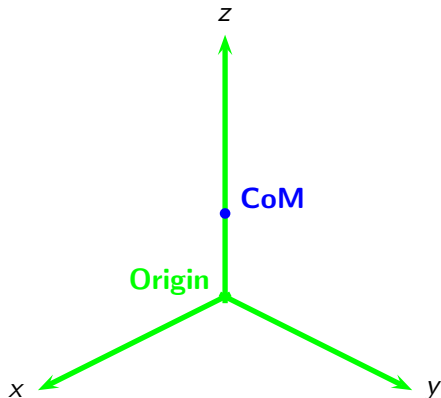
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CoM wrt. Origin	RMS of adjustment
(0,0,0.00) m	1.59 mm
(0,0,0.01) m	1.59 mm
(0,0,0.10) m	1.60 mm
(0,0,1.00) m	2.01 mm

Shifting the CoM wrt. Origin	Impact on coordinates	
	dZ	RMS
(0,0,0.00) m		
(0,0,0.01) m	0.03 mm	0.23 mm
(0,0,0.10) m	0.30 mm	2.32 mm
(0,0,1.00) m	3.00 mm	23.2 mm

# Experiment 1: Shifting the Geocenter

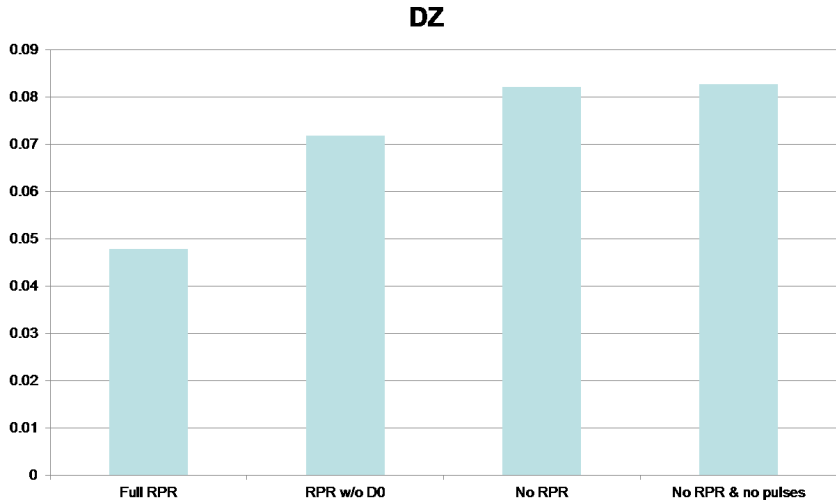


CoM wrt. Origin	RMS of adjustment
(0,0,0.00) m	1.59 mm
(0,0,0.01) m	1.59 mm
(0,0,0.10) m	1.60 mm
(0,0,1.00) m	2.01 mm

Shifting the CoM wrt. Origin	Impact on coordinates		Impact on orbits	
	dZ	RMS	dZ	RMS
(0,0,0.00) m				
(0,0,0.01) m	0.03 mm	0.23 mm	-0.53 cm	0.2 cm
(0,0,0.10) m	0.30 mm	2.32 mm	-5.23 cm	1.5 cm
(0,0,1.00) m	3.00 mm	23.2 mm	-52.1 cm	14.9 cm



# Experiment 1: Shifting the Geocenter



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## Lessons learned from Experiment 1:

- If the **ground network** is not geocentric (and the geocenter is kept fixed) **the network will be deformed**.

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- Some orbit parameters (in particular  $D_0$ ) may absorb a significant part of the geocenter shift. The amount depends on the orientation of this component (direction to the Sun) w.r.t. the orbital plane.

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- If the **ground network** is not geocentric (and the geocenter is kept fixed) **the network will be deformed**.
- Some orbit parameters (in particular  $D_0$ ) may absorb a significant part of the geocenter shift. The amount depends on the orientation of this component (direction to the Sun) w.r.t. the orbital plane.
- **The GNSS analysis system is stable and able to reconstruct the geometry between orbits and station coordinates** – even if other parameters like troposphere or (satellite) clocks have to be estimated (the ambiguities are assumed to be resolved).

# Experiment 2: Geocenter with Simulated Data

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Description of the problem

Experiment 1: Shifting the Geocenter

Experiment 2: Geocenter with Simulated Data

- The simulation setup

- Reference solutions

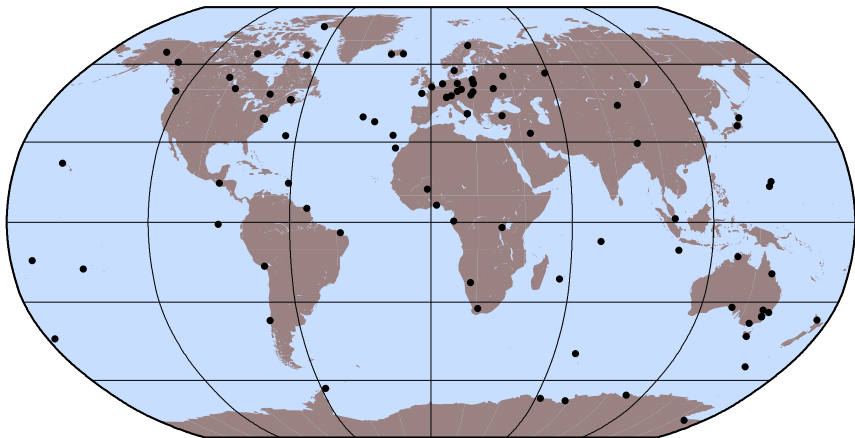
- Correlations between the parameters

Geocenter Time Series from GNSS Solution

# The simulation setup

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A network of 90 globally distributed stations has been selected:

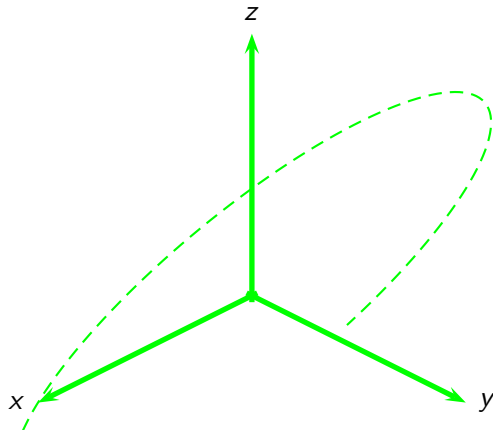


# The simulation setup

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- Geometry has been introduced from a CODE final solution.
- GPS observations have been generated for all stations.
- Code measurements without noise may be used for a solution where all ambiguities are fixed to their correct integer values.
- The standard parametrization is used for the analysis (see Experiment 1).

# Reference solutions



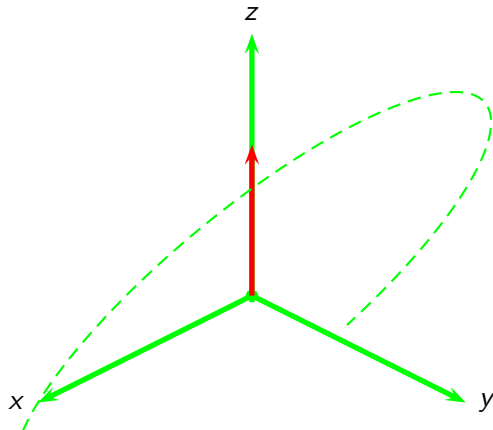
- Original geometry from simulation

Solution setup		Results			
Datum	GCC	Chi <sup>2</sup>	CRD	GCC	ORB
NNR+NNT	estimate	0.00	0.00	0.00	0.00
NNR	fixed	0.00	0.00		0.00
NNR+NNT	fixed	0.00	0.00		0.00
NNR	estimate	0.00	0.00	0.00	0.00



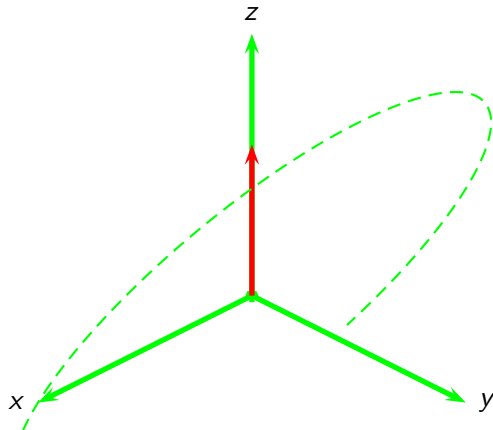
# Modified reference solutions

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- Original geometry from simulation
- Artificial geocenter shift: 10 cm

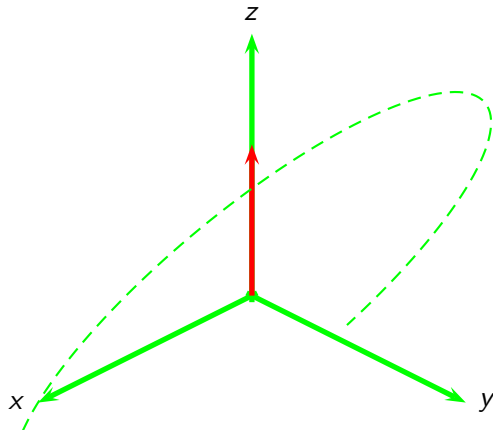
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Datum	GCC	Chi <sup>2</sup>	CRD	GCC	ORB
NNR+NNT	estimate	0.00	0.00	0.10	0.00

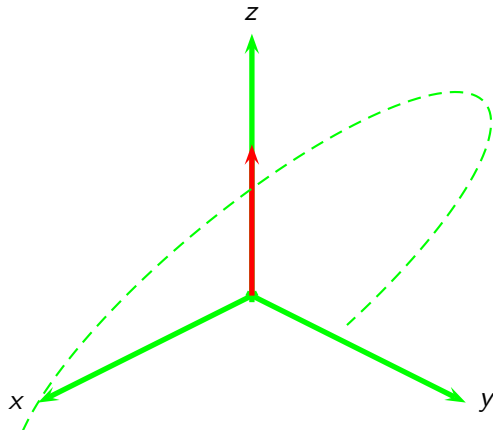
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Datum	GCC	Chi <sup>2</sup>	CRD	GCC	ORB
NNR+NNT	estimate	0.00	0.00	0.10	0.00
NNR	fixed	0.00	-0.10		0.00

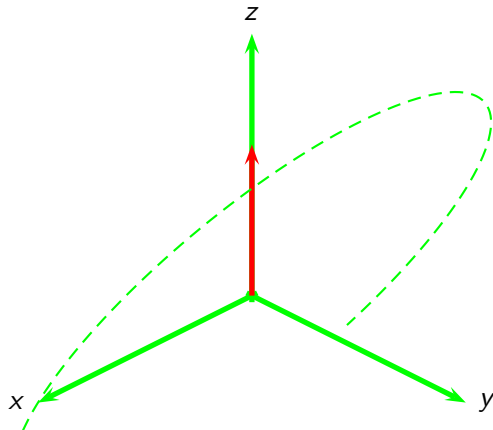
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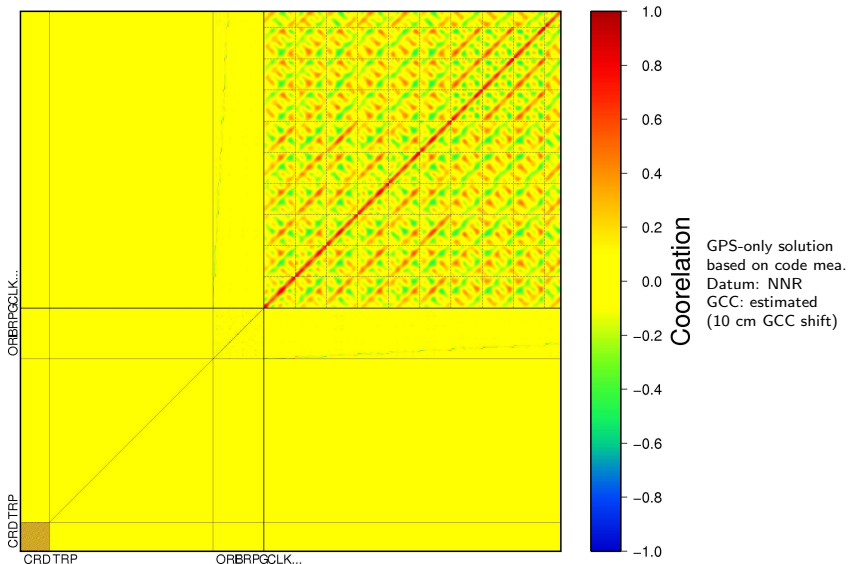
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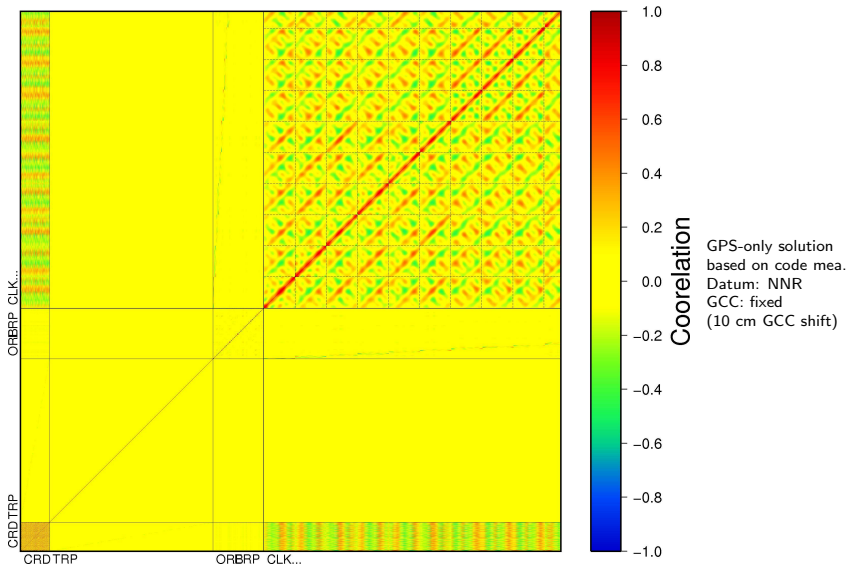
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NNR+NNT	estimate	0.00	0.00	0.10	0.00
NNR	fixed	0.00	-0.10		0.00
NNR	estimate	0.00	0.00	0.10	0.00
NNR+NNT	fixed	≠0.00	≠0.00		≠0.00

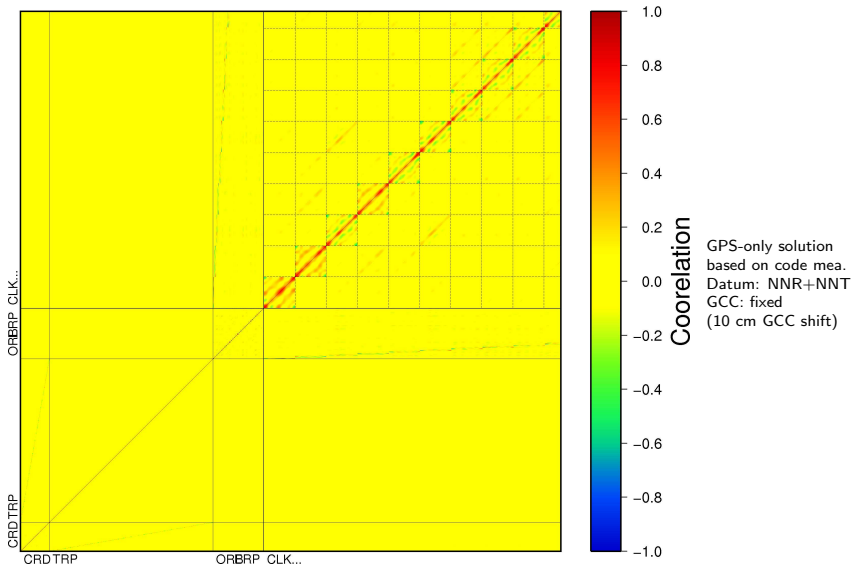
# Correlations between the parameters



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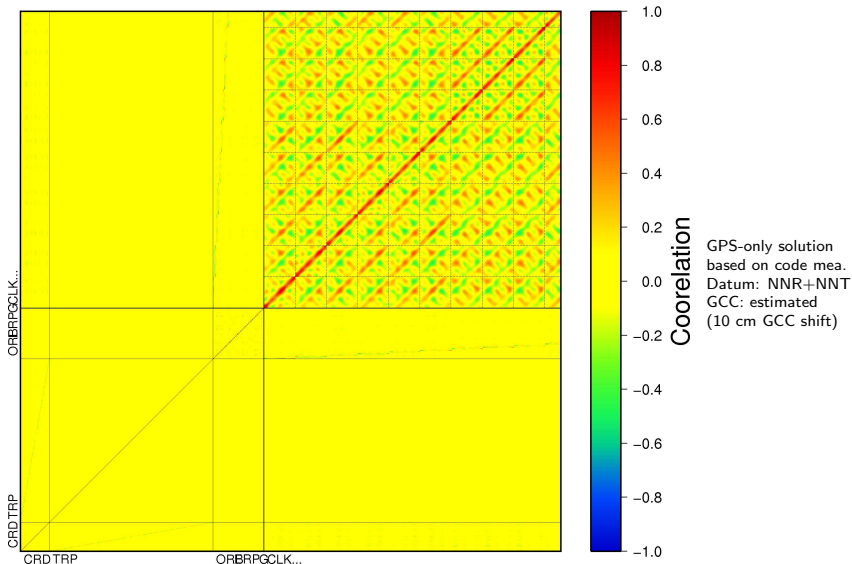


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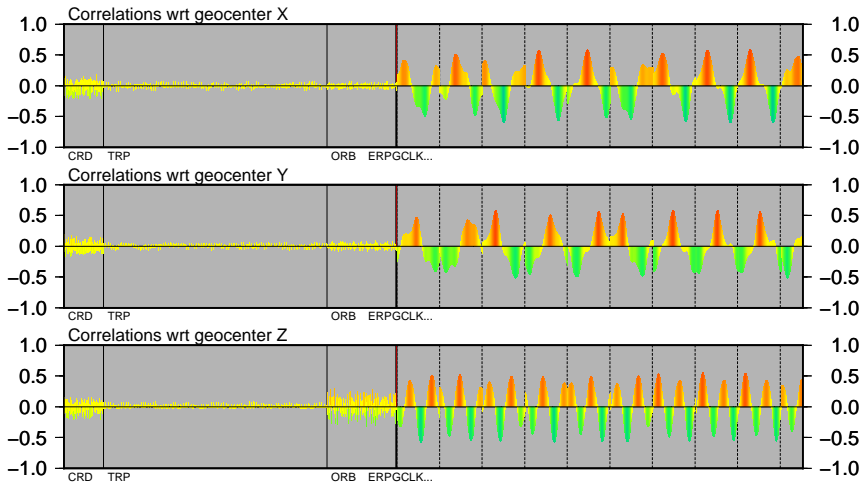




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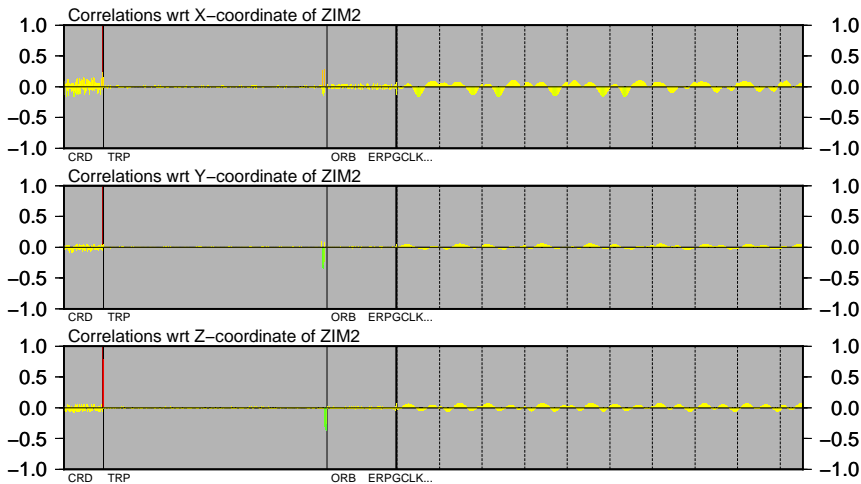


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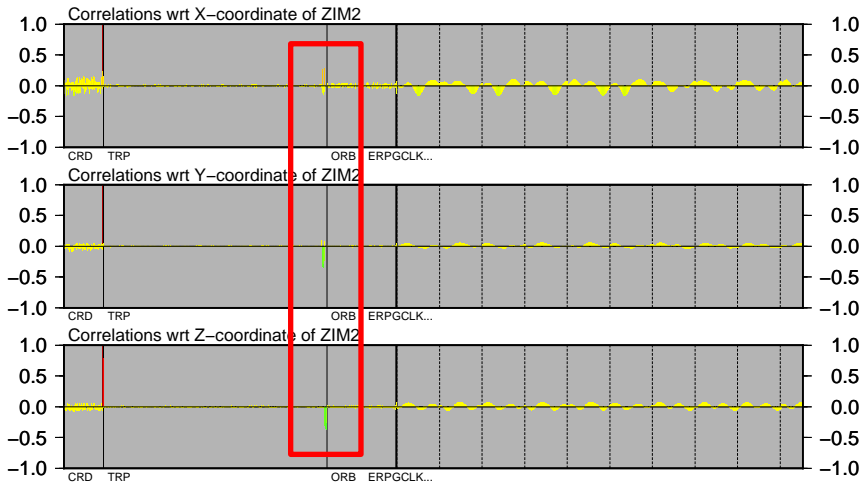
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# Experiment 2: Geocenter with Simulated Data

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## Lessons learned from Experiment 2:

- (Even) in a global solution the **NNT condition must be applied** to complete the definition of the geodetic datum.

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- (Even) in a global solution the **NNT condition must be applied** to complete the definition of the geodetic datum.
- The correlations between geocenter parameters and the satellite clock parameters derived from the a posteriori covariance matrix are of the same order of magnitude as the correlations between station height and troposphere ZPD parameters.

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⇒ In principle: **the geocenter parameters can be estimated from GNSS solutions.**

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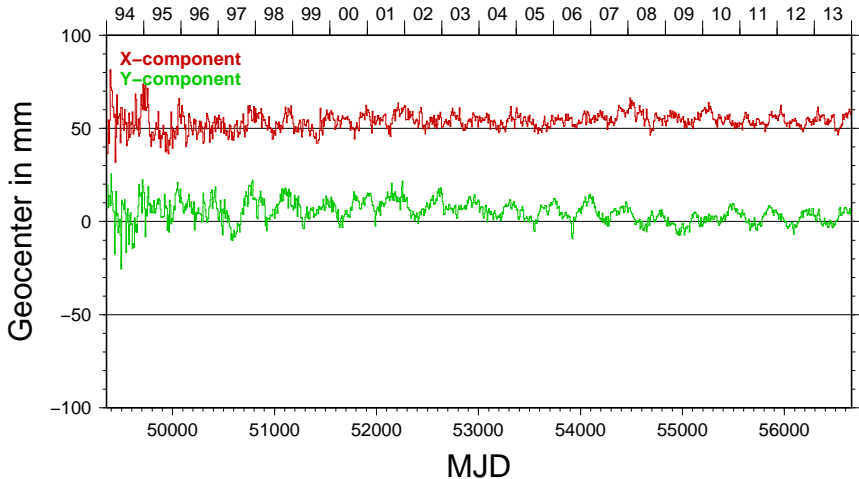
- *But what about their geodynamical interpretation?*



# Geocenter Time Series from GNSS Solution

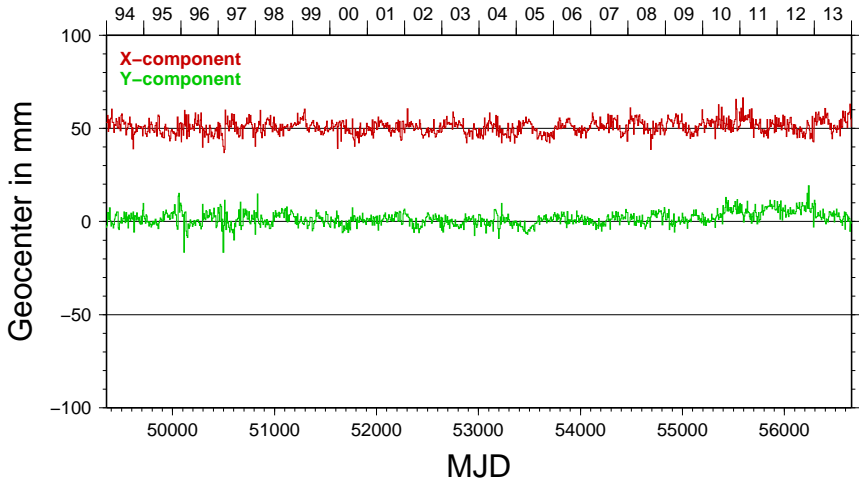
Geocenter time series from the CODE repro2

solution



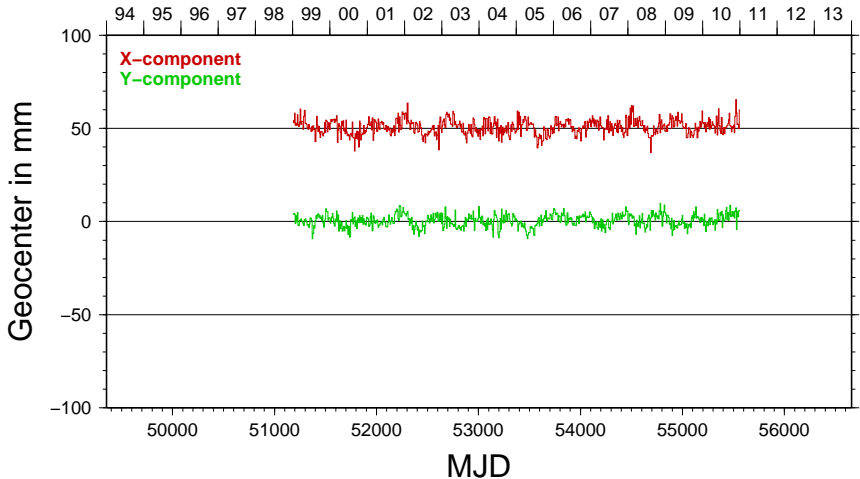
# Geocenter Time Series from GNSS Solution

Geocenter time series from a LAGEOS solution



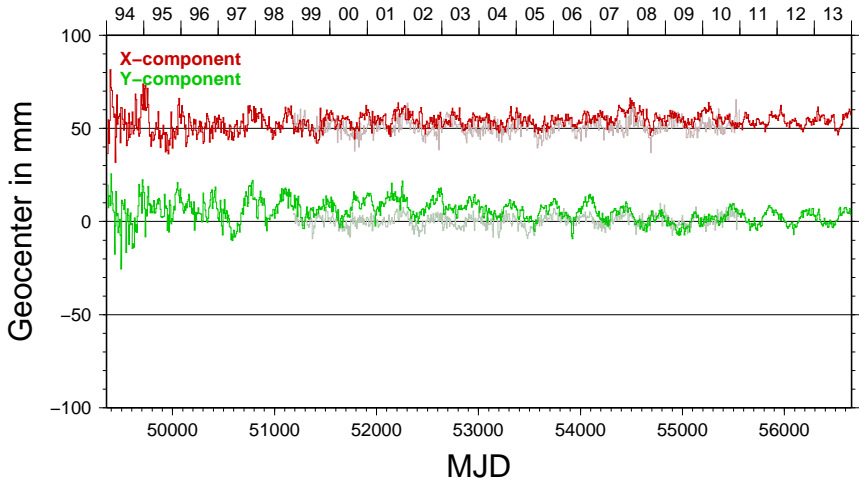
# Geocenter Time Series from GNSS Solution

Geocenter time series from a LAGEOS solution



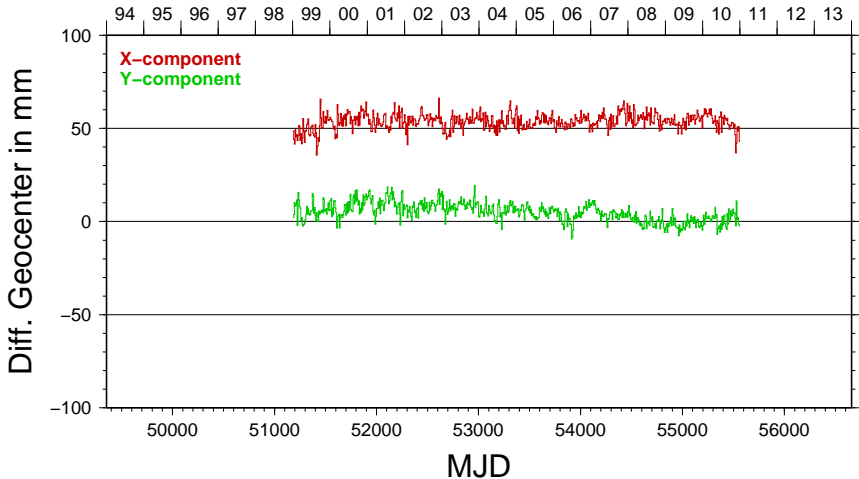
# Geocenter Time Series from GNSS Solution

Geocenter time series from the CODE repro2 and a LAGEOS solution



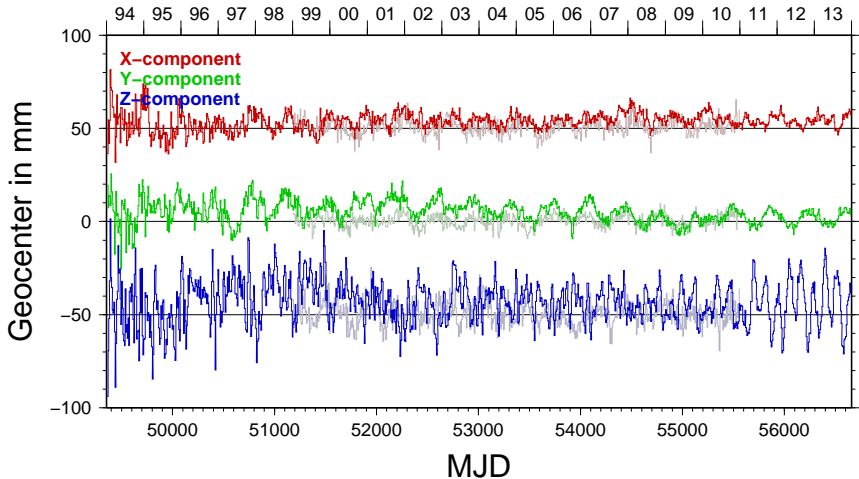
# Geocenter Time Series from GNSS Solution

Geocenter time series from the CODE repro2 and a LAGEOS solution



# Geocenter Time Series from GNSS Solution

Geocenter time series from the CODE repro2 and a LAGEOS solution



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# **Estimating the Geocenter from GNSS data**

## **Part II**

### **Orbit Modeling Reflected by Geocenter Coordinate Series**

# GNSS Orbit Modeling

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- For **GNSS satellites**, at an **altitude of ~20,000 km**,
  - ➔ non-conservative forces are very important for precise orbit determination and prediction
  - ➔ mismodelling issues or no models are used
  - ➔ gravitational forces have a low contribution to the orbit error budget
- Main non-conservative force ➔ **solar radiation pressure**
- Smaller non-conservative forces:
  - ➔ Earth radiation pressure
  - ➔ thermal radiation pressure
- Basically two types of models:
  - ➔ empirical models, based on in-orbit behavior
  - ➔ analytical/physical models, based on pre-launch information



# Solar Radiation Pressure Modeling

- Modeling of non-conservative forces is a complex task!

- Acceleration due to solar radiation pressure  $\longrightarrow \left\{ \vec{f} = -\frac{A}{M} \frac{S_0}{c} \cos \theta \left[ (1 - \rho) \vec{e}_D + 2 \left( \frac{\delta}{3} + \rho \cos \theta \right) \vec{e}_N \right] \right\}$ ,

with:

$$\alpha + \rho + \delta = 1,$$

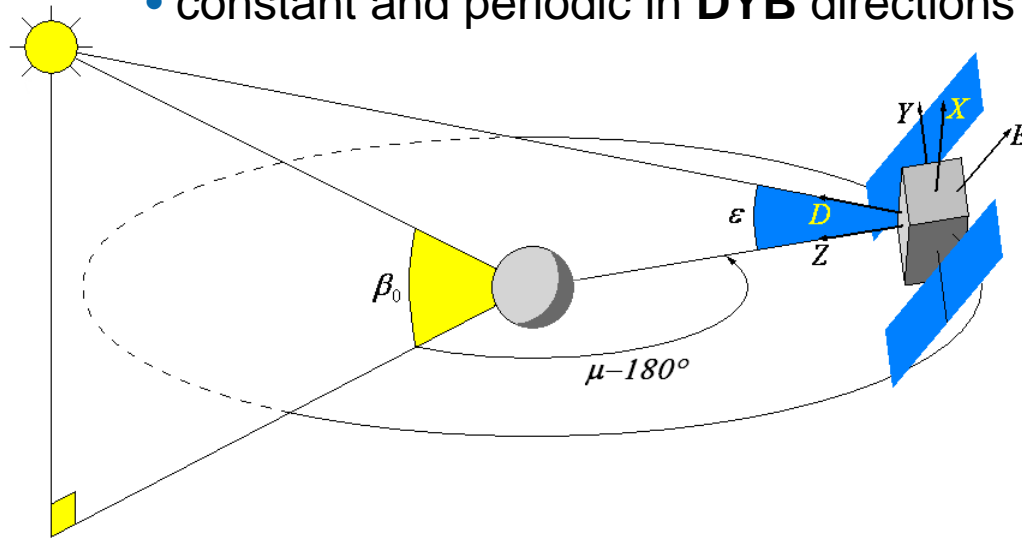
where:

- Satellite properties  $\left\{ \begin{array}{l} A \\ M \end{array} \right\}$ 
  - area of the surface
  - mass of the satellite
- Well known  $\left\{ \begin{array}{l} S_0 \\ c \end{array} \right\}$ 
  - solar irradiance at 1 AU ( $\approx 1367 \text{ W/m}^2$ )
  - velocity of light in vacuum
- $\left\{ \begin{array}{l} \alpha \\ \rho \\ \delta \end{array} \right\}$ 
  - fraction of absorbed photons
  - fraction of reflected photons
  - fraction of diffusely scattered photons
- $\left\{ \begin{array}{l} \vec{e}_D \\ \vec{e}_N \end{array} \right\}$ 
  - direction of the Sun from the satellite
  - normal to the satellite surface
- Satellite attitude, orientation in space  $\left\{ \begin{array}{l} \cos \theta = \vec{e}_D \cdot \vec{e}_N, \text{ valid only if } \cos \theta \geq 0. \end{array} \right\}$

# Solar Radiation Pressure Modeling

- **CODE empirical model:**

- 5 empirical acceleration parameters [m/s<sup>2</sup>] per arc
- constant and periodic in **DYB** directions



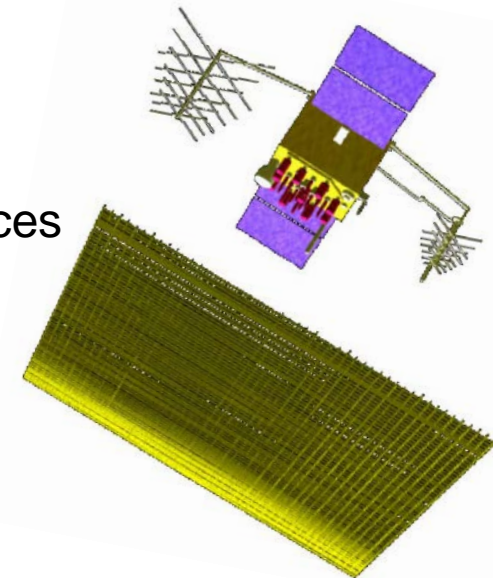
- 3 stochastic pulses per day

- radial
- along-track
- cross-track

- **Analytical models:**

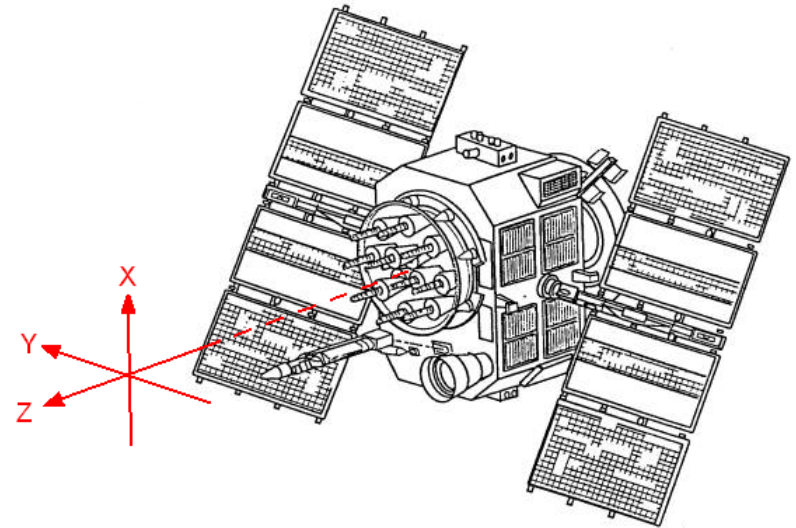
- knowledge e.g. from satellite manufacturers
- nominal attitude
- physical interaction between radiation and satellite surfaces

- Examples: **T20/T30** (Fliegel et al., 1992, 1996)  
**UCL** (Ziebart et al., 2005)



# Solar Radiation Pressure Modeling

- Physically based model:  
Simple box-wing model for SRP
- Four main surfaces:
  - Solar panels front
  - Bus +X side
  - Bus +Z side
  - Bus -Z side

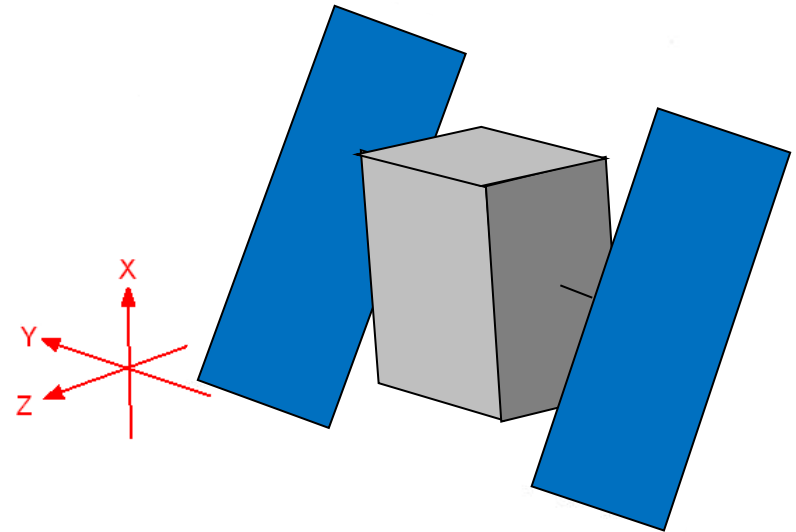


# Solar Radiation Pressure Modeling

- **Physically based model:**

- Simple box-wing model for SRP**

- Four main surfaces:
  - Solar panels front
  - Bus +X side
  - Bus +Z side
  - Bus -Z side



- Model capable of fitting the GNSS tracking data
  - ➔ adjusting the optical properties of the satellite's surfaces
- Additionally adjustment of:
  - Stochastic pulses
  - Y-bias acceleration
  - **Solar panel rotation lag angle**

# Three Different Solutions

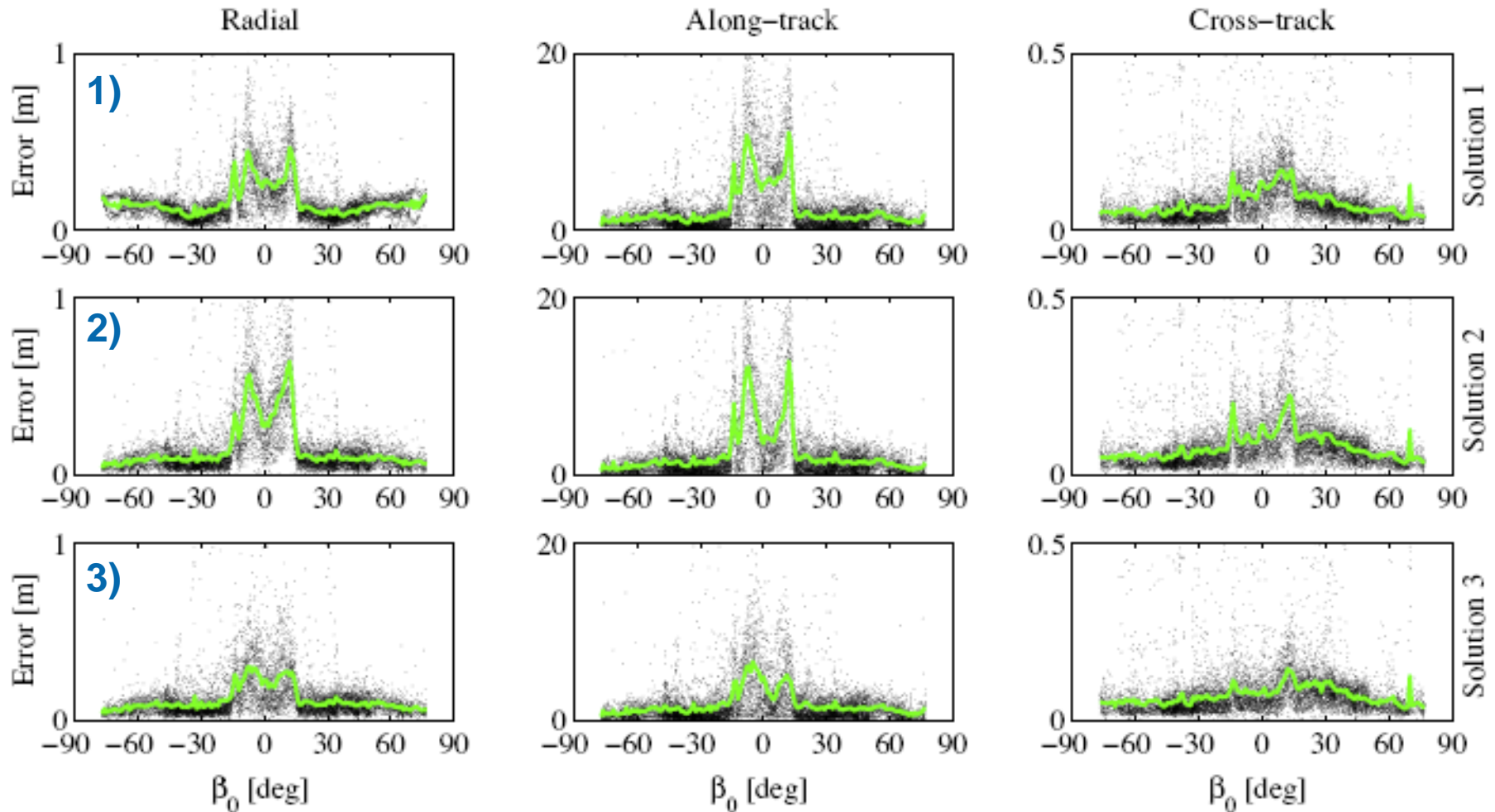
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- Reprocessing of 8 years (2004-2011) of GNSS tracking data
  - ➔ 3 solutions differing only on the non-conservative force modeling
  - ➔ GPS+GLONASS global solutions (up to 254 ground stations used)
- Solutions:
  - 1) CODE (5-parameter) model + nominal yaw attitude (Beutler et al. 1994)
  - 2) Adjustable box-wing model + nominal yaw attitude (Rodriguez-Solano et al. 2012)
  - 3) Adjustable box-wing model + yaw attitude models (Rodriguez-Solano et al. 2013)
    - ➔
      - GPS-IIA (Bar-Sever 1996)
      - GPS-IIR (Kouba 2009)
      - GLONASS-M (Dilssner et al. 2011)
- Following results from:

Rodriguez-Solano CJ, Hugentobler U, Steigenberger P, Bloßfeld M, Fritsche M (2014) **Reducing the draconitic errors in GNSS geodetic products**. Journal of Geodesy 8(6): 559-574, doi:10.1007/s00190-014-0704-1

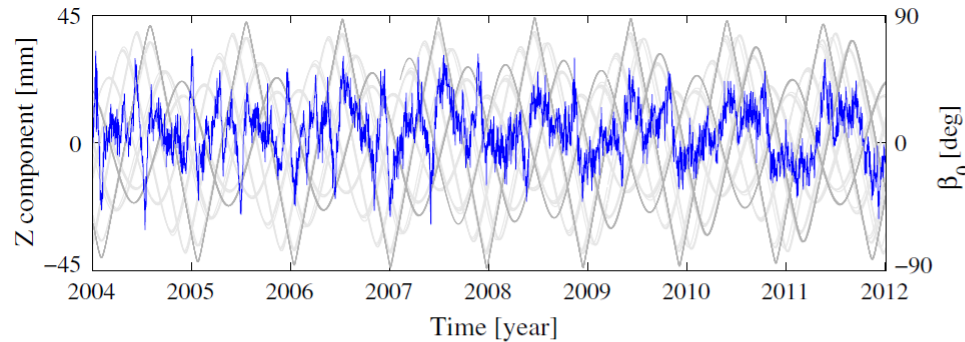
# Impact on Satellite Orbits

- Orbit prediction error for Block IIA vs Sun elevation above the orbital plane

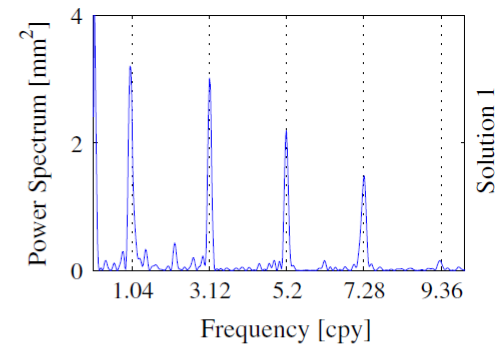


# Impact on the Geocenter Z-component

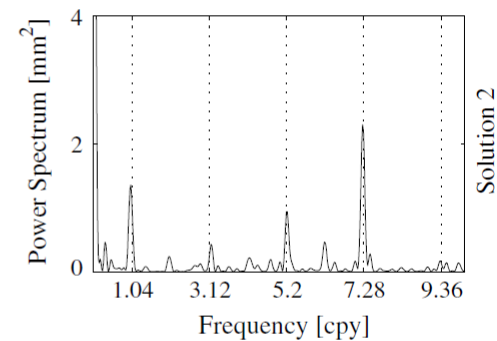
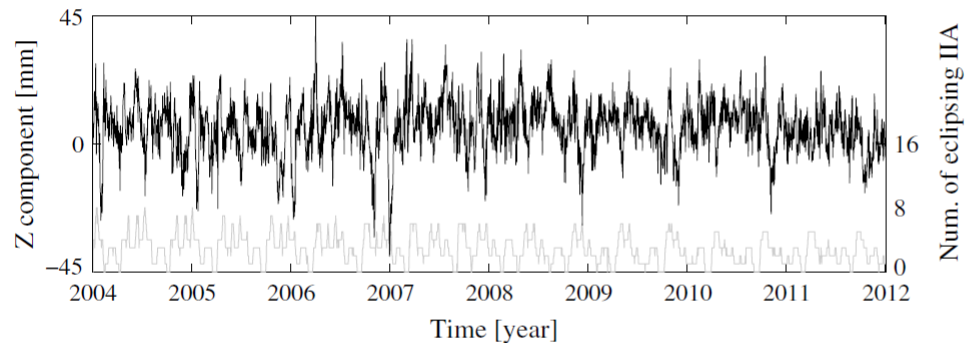
Time series



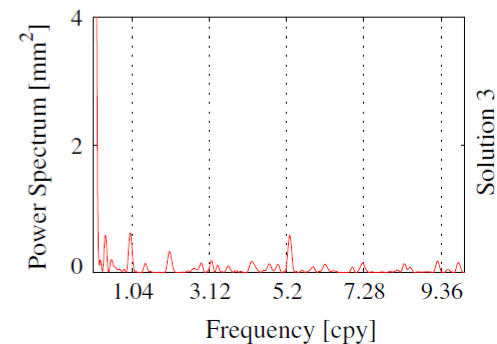
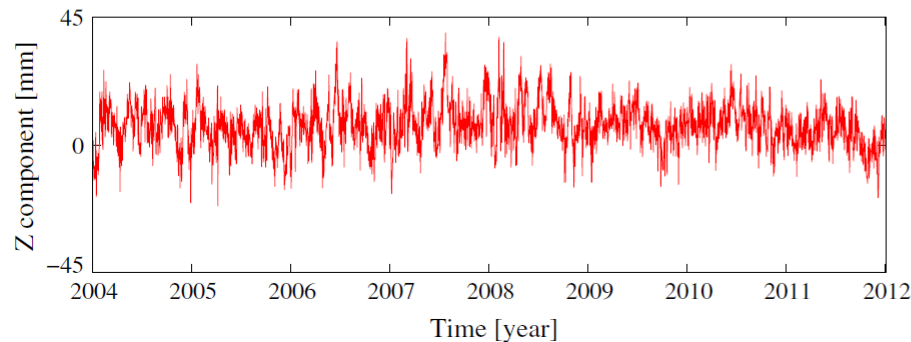
Draconitic harmonics



**CODE +  
Nominal  
Attitude**



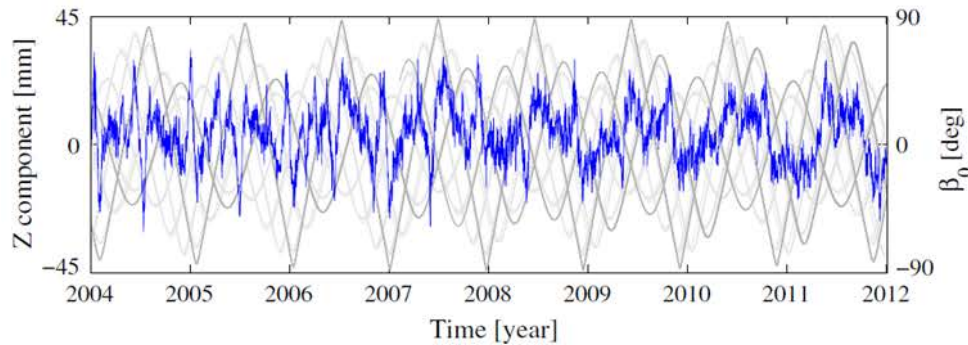
**Box-wing +  
Nominal  
Attitude**



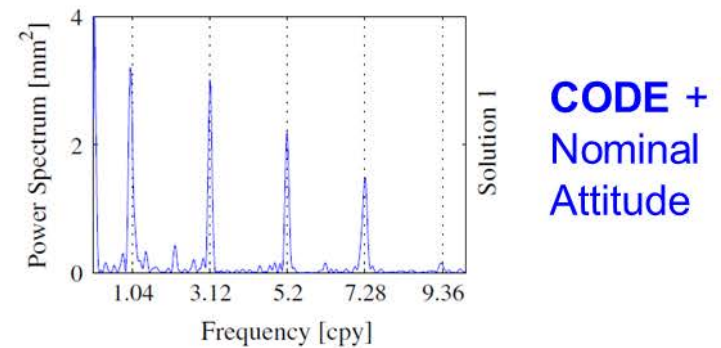
**Box-wing +  
Yaw Attitude  
Modeled**

# Impact on the Geocenter Z-component

Time series



Draconitic harmonics

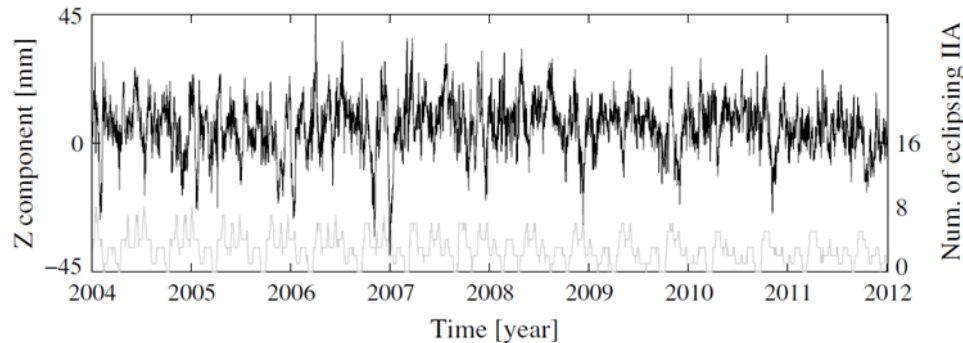


- Why the CODE model (solution 1) shows mainly odd draconitic harmonics?  
→ not yet an explanation

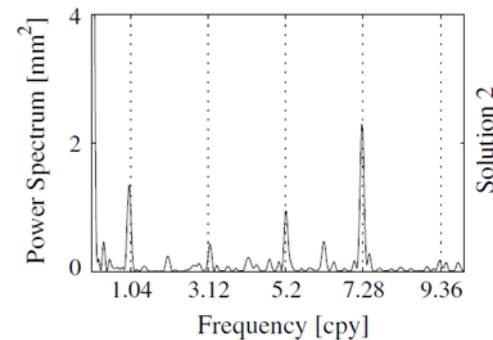


# Impact on the Geocenter Z-component

Time series



Draconitic harmonics

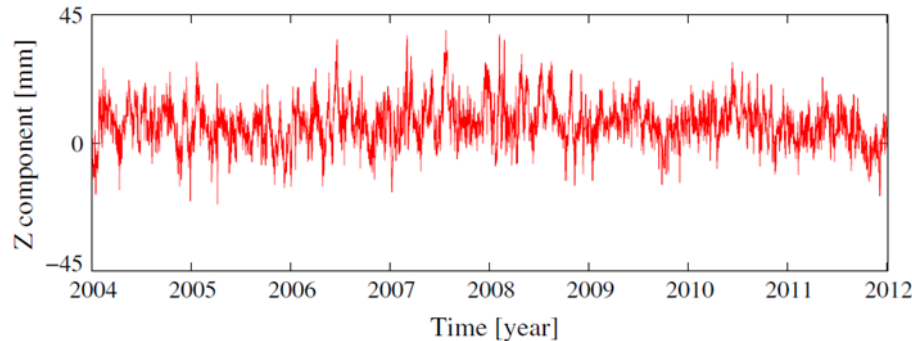


**Box-wing +  
Nominal  
Attitude**

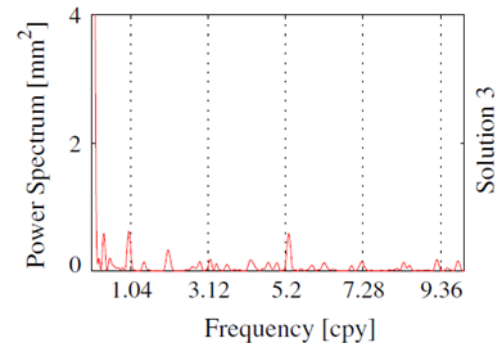
- Why the box-wing model with nominal attitude (solution 2) shows mainly errors at the 7<sup>th</sup> draconitic harmonic? → explanation:
  - The box-wing model with nominal attitude shows a degradation in the orbits (compared to the CODE model) during eclipse seasons, especially for GPS-IIA satellites
  - The differences in days between consecutive GPS orbital planes along the ecliptic (not the equator) shows a peak close to 50 days → 7<sup>th</sup> draconitic harmonic

# Impact on the Geocenter Z-component

Time series



Draconitic harmonics



**Box-wing +  
Yaw Attitude  
Modeled**

- Why the box-wing model combined with the yaw attitude models (solution 3) reduces significantly the 7<sup>th</sup> draconitic harmonic? → explanation:
  - The use of the yaw attitude models shows a significant improvement in the orbits (compared to the two previous models) during eclipse seasons, especially for GPS-IIA satellites

# Conclusions

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- Geocenter Z-component draconitic errors:  
→ **In total 92% reduction** from solution 1 to solution 3
- Despite a large reduction of the draconitic errors obtained for the geocenter Z-component → not yet obtained the expected geophysical annual signal
- The geocenter Z-component is very sensitive to orbit modeling errors
- The box-wing model combined with the yaw attitude models does not remove completely the draconitic errors in the GNSS orbits  
→ other modeling problems remain, especially during eclipse seasons
- How the geocenter Z-component time series would look like if the remaining draconitic errors in the GNSS orbits could be corrected?